Section 7.4 Solutions

2. a) $\dfrac{x}{x^2+x-2} = \dfrac{A}{x+2} + \dfrac{B}{x-1}$

b) $\dfrac{x^2}{x^2+x+2} = 1 - \dfrac{x+2}{x^2+x+2}$ after long division

10. $\dfrac{y}{(y+4)(2y-1)} = \dfrac{A}{y+4} + \dfrac{B}{2y-1}$ or $y = A(2y-1) + B(y+4)$

when $y = -4$, $-4 = -9A \implies A = 4/9$

$y = \frac{1}{2}$, $\frac{1}{2} = \frac{9}{2}B \implies B = \frac{1}{9}$

$\therefore \displaystyle\int \frac{y}{(y+4)(2y-1)}\,dy = \int \left(\frac{4}{9}\cdot\frac{1}{y+4} + \frac{1}{9}\cdot\frac{1}{2y-1}\right)dy$

$= \boxed{\frac{4}{9}\ln|y+4| + \frac{1}{18}\ln|2y-1| + C}$

12. $\dfrac{x-4}{(x^2-5x+6)} = \dfrac{A}{(x-3)} + \dfrac{B}{(x-2)}$ or $x-4 = A(x-2) + B(x-3)$

when $x = 3$, $-1 = A \implies A = -1$

$x = 2$, $-2 = -B$, $B = 2$

$\therefore \displaystyle\int_0^1 \frac{x-4}{x^2-5x+6}\,dx = \int_0^1 \left(-\frac{1}{x-3} + \frac{2}{x-2}\right)dx = \Big[-\ln|x-3| + 2\ln|x-2|\Big]_0^1$

$= \boxed{\ln\frac{3}{8}}$ after some algebra

20. $\dfrac{x^2-5x+16}{(2x+1)(x-2)^2} = \dfrac{A}{2x+1} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$ (or)

$x^2-5x+16 = A(x-2)^2 + B(2x+1)(x-2) + C(2x+1)$

$x = -\frac{1}{2}$: $\frac{25}{4} = \frac{25}{4}A \implies A = 3$

$x = 2$: $10 = 5C \implies C = 2$

$x = 0$: $16 = 3(-2)^2 + B(1)(-2) + 2(1)$

$16 = 12 - 2B + 2$

$2 = -2B \implies B = -1$

$\displaystyle\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx = \int \left(\frac{3}{2x+1} - \frac{1}{x-2} + \frac{2}{(x-2)^2}\right)dx = \boxed{\frac{3}{2}\ln|2x+1| - \ln|x-2| - \frac{2}{x-2} + C}$

26. $$\frac{x^2+x+1}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$$

or $$x^2+x+1 = (Ax+B)(x^2+1) + (Cx+D)$$
$$= Ax^3 + Bx^2 + (A+C)x + (B+D)$$ after multiplying out the RHS

Equating coefficients, we see that $A=0$, $B=1$, $C=1$, $D=0$

$$\int \frac{x^2+x+1}{(x^2+1)^2}\,dx = \int \frac{1}{x^2+1}\,dx + \int \frac{x}{(x^2+1)^2}\,dx$$
$$= \boxed{\tan^{-1}x - \frac{1}{2(x^2+1)} + C}$$

32. $$\frac{x}{x^2+4x+13} = \frac{x}{(x+2)^2+9}$$ after completing the square in the denominator.

$$\int_0^1 \frac{x}{(x+2)^2+9}\,dx = \int_2^3 \frac{u-2}{u^2+9}\,du$$, where $u = x+2$, $du = dx$

x	u
0	2
1	3

$$= \int_2^3 \frac{u}{u^2+9}\,du - 2\int_2^3 \frac{1}{u^2+9}\,du$$
$$= \frac{1}{2}\left[\ln|u^2+9|\right]_2^3 - \frac{2}{3}\left[\tan^{-1}\left(\frac{u}{3}\right)\right]_2^3$$
$$= \frac{1}{2}\left[\ln 18 - \ln 13\right] - \frac{2}{3}\left[\tan^{-1}(1) - \tan^{-1}\left(\frac{2}{3}\right)\right]$$
$$= \ln\sqrt{\frac{18}{13}} - \frac{2}{3}\left(\frac{\pi}{4}\right) + 2\tan^{-1}\left(\frac{2}{3}\right)$$
$$= \boxed{\ln\sqrt{\frac{18}{13}} - \frac{\pi}{6} + \frac{2}{3}\tan^{-1}\left(\frac{2}{3}\right)}$$